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Extension of Some Common Fixed Point Theorems using Compatible Mappings in Fuzzy Metric Space

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Abstract: In this paper we have proved some common Fixed Point theorems for four mappings using the notion of compatibility.
Keywords: Fuzzy Metric Space, Compatible Mappings

I. Introduction

The concept OF Fuzzy sets was investigated by Zadeh [1]. Here we are dealing with the fuzzy metric space defined by Kramosil and Michalek [2] and modified by George and Veeramani [3]. Grabiec[4] has also proved fixed point results for fuzzy metric space with different mappings. Singh and Chauhan[5] gave the results using the concept of compatible mappings in Fuzzy metric space. Jungck [6] introduced the concept of compatible mapping of type (A) and type (B)In fuzzy metric space. Singh and jain [7] proved the fixed point theorems in fuzzy metric space using the concept of compatibility and semicompatibily. Sharma[8] also done work on compatible mappings.

II. FUZZY METRIC SPACE

Definition[2]: A 3-tuple (X,M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty]$ satisfying the following conditions

(f1) $M(x, y, t) > 0$
(f2) $M(x, y, t) = 1$ if and only if $x = y$
(f3) $M(x, y, t) = M(y, x, t)$;
(f4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(f5) $M(x, y, .) : (0, \infty) \to (0, 1]$ is continuous.
$x,y,z \in X$ and $t,s > 0$

Then M is called a fuzzy metric on X. Then M(x, y, t) denotes the degree i.e. of nearness between x and y with respect to t.

Compatible and Non compatible mappings: Let A and S be mapping from a fuzzy metric space (X,M, *) into itself.Then the mappings are said to be compatible if

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1, \forall t > 0,$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} x \in X$$

from the above definition it is inferred that A and S are non compatible maps from a fuzzy metric space (X,M,*) into itself if

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x \in X$$

but either

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) \ne 1,$$ or the limit does not exist.

Main Results:

Theorem:-Let A,B,S,T be self maps of complete fuzzy metric space (X,M,*) such that a*b = min(a,b) for some y in X.

(a) $A(X) \subset T(X), B(X) \subset S(X), T(Y) \subset A(Y)$
(b) S and T are continuous.

(c) [A,S],[B,T] are compatible pairs of maps
(d) For all x,y in X ,$k \in (0,1)$,t> 0.
$M(Ax,By,KT) \geq \min \{ M(Sx,Ty,t),M(Ax,Sx,t),M(By,Ty,t),M(By,Sx,t),M(Ax,Ty,t),M(Ay,Tx,t)\}$
For all $x,y \in X$ LIM $n \to \infty$ $M(x,y,t) \to 1$ then A,B,S,T have a common fixed point in X.

Proof:- Let x_0 be an arbitrary point inX. Construct a sequence $\{y_n\}$ in X such that $y_{2n-1}= Tx_{2n-1}=Ax_{2n-2}$
And $y_{2n}= Sx_{2n}=Bx_{2n-1}=Tx_{2n}$, for n= 0,1,2......
Put $x= x_{2n,}$ $y= x_{2n+1}$
$M(y_{2n+1},y_{2n+2},kt) = M (Ax_{2n},Bx_{2n+1},kt)$
$\geq \min \{(M(Sx_{2n}, Tx_{2n+1},t),M((Ax_{2n},Sx_{2n},t),M(Bx_{2n+1},Tx_{2n+1},t),M(Tx_{2n},Ax_{2n+1},t),M(Ax_{2n},Tx_{2n+1},t),M(Bx_{2n+1}, Sx_{2n},t)\}$
$\geq \min\{M(y_{2n,}y_{2n+1},kt),M(y_{2n+1},y_{2n+2,}t),1\}$
Which implies
$\{M(y_{2n+1,}y_{2n+2},kt) \geq M(y_{2n},y_{2n+1,}t)$.
In general
$\{M(y_{n,}y_{n+1},kt) \geq M(y_{n-1},y_n,t)$ (1)
To prove that $\{y_n\}$ is a Cauchy sequence we will prove (b) is true for all $n \geq n_0$ and every $m \in N$
$\{M(y_{n,}y_{n+m},t) > 1 - \lambda$ (2)
Here we use induction method
From(1) we have
$M(y_{n,}y_{n+1},t) \geq M(y_{n-1},y_{n,}t /k) \geq M((y_{n-2},y_{n-1,}t/k^2) \geq \ldots\ldots.. \geq M(y_0,y_{1,}t/k^n) \to 1$ as $n \to \infty$
i.e for t> 0, $\lambda \in (0,1)$. We can choose $n_0 \in N$, such that
$\{M(y_{n,}y_{n+1},t) > 1 - \lambda$ (3)
Thus (2) is true for m=1.Suppose (2) is true for m then will show that it is true for m+1. By the definition of fuzzy metric space, we have
$M(y_{n,}y_{n+m+1},t) \geq \min\{M(y_{n,}y_{n+m},t/2),M(y_{n+m,}y_{n+m+1},t/2)\} > 1 - \lambda$
Hence(2) is true for m+1.Thus $\{y_n\}$ is a Cauchy sequence.By completeness of (X,M,*) ,$\{ y_n\}$ Converge (
Using (3), we have $M(ASx_{2n},SAx_{2n} ,t/2) \to 1$
$M(SAx_{2n},Sz,t) \geq \min\{M(ASx_{2n},SAx_{2n} ,t/2), M(SAx_{2n},Sz,t/2) \}> 1 - \lambda$
For all $n \geq n_0$
Hence $ASx_{2n} \to Sz = TSx_{2n}$ (4)
Similarly
$BTx_{2n-1} \to Tz = ATx_{2n-1}$ (5)
Now put $x= Sx_{2n}$ and $y = Tx_{2n-1}$

$M(ASx_{2n}, BTx_{2n-1},kt) \geq \min \{ M (S^2x_{2n}, T^2x_{2n-1},t),M(ASx_{2n}, S^2x_{2n},t),M(BTx_{2n-1}, T^2x_{2n-1},t),M(BTx_{2n-1}, S^2x_{2n},t),M(ASx_{2n}, T^2x_{2n-1},t),M(TSx_{2n},ATx_{2n-1},t)\}$
Taking limit as $n \to \infty$ and using (4) and (5)
We get $M(Sz,Tz,kt) \geq M(Sz,Tz,t)$, which implies
Sz = Tz (6)
Now put x=y and $y= Tx_{2n-1}$
$M(Ay,BTx_{2n-1},kt) \geq \min\{M(Sy, T^2x_{2n-1},t),M(Ay,Sy,t),M(BTx_{2n-1},Sy,t),M(Ay, T^2x_{2n-1},t),M(Ty,ATx_{2n-1},T)\}$
Taking the limit as $n \to \infty$ and using (5) and (6) we get
Az=Tz (7)
Now using (6) and (7)
$M(Az,Bz,kt) \geq \min\{M(Sz,Tz,t),M(Az, Sz,t),M(Bz,Tz,t),M(Bz,Sz,t),M(Az,Tz,t),M(Az,Tz,t)$
$= \min\{M(Tz,Tz,t),M(Az,Az,t),M(Az,Bz,t),M(Az,Bz,t),M(Az,Az,t),M(Az,Bz,t)\}$
$\geq M(Az,Bz,t)$
Which implies Az=Bz
Using (6),(7) and (8)
We get
Az=Bz=Sz=Tz
Now
$M(Ax_{2n},Bz,kt) \geq \min\{ M(Sx_{2n},Tz,t),M(Ax_{2n},Sx_{2n},t),(Bz,Tz,t),M(Bz,Sx_{2n},t),M(Ax_{2n},Tz,t),M(Tx_{2n},Az,t)\}$
Taking the limit as $n \to \infty$ and using (9) we get
Z =Bz
Thus z is common fixed point of A,B,S,T.
For uniqueness let w be another common fixed point then we have

$M(Az,Bw,kt) \geq \min\{M(Sz,Tw,t),M(Az,Sz,t),M(Bw,Tw,t),M(Bw,Sw,t),M(Az,Tw,t),M(Tz,Aw,t)\}$
i.e. $M(z,w,kt) \geq M(z,w,t)$
hence z =w this completes the proof.

III. Conclusion

Here we proved the theorem using the notion of compatibility without exploiting the condition of t-norm.

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